



# **Cambridge International AS & A Level**

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## **FURTHER MATHEMATICS**

**9231/11**

Paper 1 Further Pure Mathematics 1

**May/June 2024**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 The cubic equation  $2x^3 + x^2 - px - 5 = 0$ , where  $p$  is a positive constant, has roots  $\alpha, \beta, \gamma$ .

(a) State, in terms of  $p$ , the value of  $\alpha\beta + \beta\gamma + \gamma\alpha$ .

[1]

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(b) Find the value of  $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ .

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- (c) Deduce a cubic equation whose roots are  $\alpha\beta, \beta\gamma, \alpha\gamma$ .

[1]

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- (d) Given that  $\alpha^2 + \beta^2 + \gamma^2 = \frac{1}{3}$ , find the value of  $p$ .

[2]

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- 2 Prove by mathematical induction that  $6^{4n} + 38^n - 2$  is divisible by 74 for all positive integers  $n$ . [6]

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- 3 (a) Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^N r(r+1)(3r+4) = \frac{1}{12}N(N+1)(N+2)(9N+19). \quad [3]$$

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- (b)** Express  $\frac{3r+4}{r(r+1)}$  in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^N \frac{3r+4}{r(r+1)} \left(\frac{1}{4}\right)^{r+1}$$

in terms of  $N$ .

[4]

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- (c)** Deduce the value of  $\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)} \left(\frac{1}{4}\right)^{r+1}$ . [1]

- 4 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & 1 \end{pmatrix}$ .

(a) The matrix  $\mathbf{M}$  represents a sequence of two geometrical transformations in the  $x$ - $y$  plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(b) Write  $\mathbf{M}^{-1}$  as the product of two matrices, neither of which is  $\mathbf{I}$ .

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- (c) Find the equations of the invariant lines, through the origin, of the transformation represented by  $\mathbf{M}$ . [5]

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- (d) The triangle  $ABC$  in the  $x$ - $y$  plane is transformed by  $\mathbf{M}$  onto triangle  $DEF$ .

Given that the area of triangle  $DEF$  is  $28 \text{ cm}^2$ , find the area of triangle  $ABC$ . [2]

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- 5 The points  $A$ ,  $B$ ,  $C$  have position vectors

$$2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}, \quad 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}, \quad -3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k},$$

respectively, relative to the origin  $O$ .

- (a) Find the equation of the plane  $ABC$ , giving your answer in the form  $ax + by + cz = d$ .

[5]

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The point  $D$  has position vector  $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ .

- (b) Find the perpendicular distance from  $D$  to the plane  $ABC$ .

[2]

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- (c) Find the shortest distance between the lines  $AB$  and  $CD$ .

[5]

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6 The curve  $C$  has equation  $y = \frac{x^2 + ax + 1}{x + 2}$ , where  $a > \frac{5}{2}$ .

- (a) Find the equations of the asymptotes of  $C$ . [3]

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- (b) Show that  $C$  has no stationary points. [4]

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- (c) Sketch  $C$ , stating the coordinates of the point of intersection with the  $y$ -axis and labelling the asymptotes. [3]

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(d) (i) Sketch the curve with equation  $y = \left| \frac{x^2 + ax + 1}{x + 2} \right|$ . [2]

(ii) On your sketch in part (i), draw the line  $y = a$ . [1]

(iii) It is given that  $\left| \frac{x^2 + ax + 1}{x + 2} \right| < a$  for  $-5 - \sqrt{14} < x < -3$  and  $-5 + \sqrt{14} < x < 3$ .

Find the value of  $a$ . [2]

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- 7 The curve  $C$  has polar equation  $r^2 = (\pi - \theta) \tan^{-1}(\pi - \theta)$ , for  $0 \leq \theta \leq \pi$ .

(a) Sketch  $C$  and state the polar coordinates of the point of  $C$  furthest from the pole.

[3]

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(b) Using the substitution  $u = \pi - \theta$ , or otherwise, find the area of the region enclosed by  $C$  and the initial line.

[7]

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- (c) Show that, at the point of  $C$  furthest from the initial line,

$$2(\pi - \theta)\tan^{-1}(\pi - \theta)\cot\theta - \frac{\pi - \theta}{1 + (\pi - \theta)^2} - \tan^{-1}(\pi - \theta) = 0$$

and verify that this equation has a root for  $\theta$  between 1.2 and 1.3.

[5]

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**Additional page**

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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